**Hypothesis Testing**

Recall that the sample mean has a distribution with:

1. Mean . That is, when we take all possible samples of size *n* from a population, and calculate the mean of each of those samples, we will see that the mean of all those means, , will be equal to the population mean . If we assume that the sample comes from the population with mean (i.e., the value of assumed to be true under the null hypothesis), then the distribution of the sample means, , will have mean .
2. Standard deviation . That is,, the standard deviation of the sample mean distribution, is equal to the standard deviation of the population, , divided by the square root of the sample size *n*.

So, when doing a hypothesis test, using the statistic is essentially taking the statistic, subtracting its mean , and dividing the difference by the standard deviation of , . This is called standardizing the variable , i.e., looking at how many standard deviations the value is from , the mean that is assumed to have under the null hypothesis. For our intents and purposes, this standardized variable will have either a *z-* or a *t-* distribution (see cases 1-3 below).

The hypothesis testing procedure is described below. Note the following:

1. Before running the hypothesis test, decide on an acceptable level of α, probability of Type I Error. That is, set the acceptable threshold for α, the probability of rejecting the null hypothesis when it is actually true, to be 0.1, 0.05, 0.01, etc, *a priori*.
2. In all 3 cases below, the null hypothesis is **H0: .**
3. Recall that you cannot *accept* or *prove* a hypothesis. Your only two options are 1) rejecting H0 for Ha or 2) failing to reject H0 for Ha.
4. On the next page, CLT stands for Central Limit Theorem.

***Case 1: Normal population, known***

*Relatively unrealistic situation, because is unknown in most cases. That is, to know , we generally need to know the population mean μ first*. *But if we know μ, there’s no need for hypothesis testing about this parameter.*

*Test statistic:*  = **z**

***Possible Alternative Hypotheses/Rejection Regions/P-Values***

1. **Ha: *upper-tailed test***

*Rejection Region:*

*Obtain with:*

*=ABS(NORMSINV(α)) Excel*

*abs(qnorm(α)) R*

*P-value:*

*Obtain P-value with:*

*=1-NORMSDIST(z) Excel*

*1-pnorm(z) R*

1. **Ha: *lower-tailed test***

*Rejection Region:*

*Obtain with:*

*=NORMSINV(α) Excel*

*qnorm(α) R*

*P-value:*

*Obtain P-value with:*

*=NORMSDIST(-1\*ABS(z)) Excel*

*pnorm(-1\*abs(z)) R*

1. **Ha: *two-tailed test***

*Rejection Region:*  OR

*Obtain with:*

*=ABS(NORMSINV(α/2)) Excel*

*abs(qnorm(α/2)) R*

*P-Value:* ]

*Obtain P-value with:*

*=2\*(1-NORMSDIST(ABS(z))) Excel*

*2\*(1-pnorm(abs(z))) R*

***Case 2: Large n (≥40), unknown***

*N is large, so CLT applies; X can have any distribution. With large n, S is close to , so plugging in S for doesn’t add much extra variability in the denominator of the test statistic and it still has a z-distribution.*

*Test statistic:*  = **z**

***Possible Alternative Hypotheses/Rejection Regions/P-Values***

1. **Ha: *upper-tailed test***

*Rejection Region:*

*Obtain with:*

*=ABS(NORMSINV(α)) Excel*

*abs(qnorm(α)) R*

*P-value:*

*Obtain P-value with:*

*=1-NORMSDIST(z) Excel*

*1-pnorm(z) R*

1. **Ha: *lower-tailed test***

*Rejection Region:*

*Obtain with:*

*=NORMSINV(α) Excel*

*qnorm(α) R*

*P-value:*

*Obtain P-value with:*

*=NORMSDIST(-1\*ABS(z)) Excel*

*pnorm(-1\*abs(z)) R*

1. **Ha: *two-tailed test***

*Rejection Region:*  OR

*Obtain with:*

*=ABS(NORMSINV(α/2)) Excel*

*abs(qnorm(α/2)) R*

*P-Value:* ]

*Obtain P-value with:*

*=2\*(1-NORMSDIST(ABS(z))) Excel*

*2\*(1-pnorm(abs(z))) R*

***Case 3: Normal pop., small n (<40), unknown***

*Notes: N is small, so CLT won’t apply. With small n, S is no longer close to σX. Therefore the denominator of the test statistic below adds extra variability and won’t have a z-distribution, but rather a t-distribution.*

*Test statistic:*  = **t**

***Possible Alternative Hypotheses/Rejection Regions/P-Values***

1. **Ha: *upper-tailed test***

*Rejection Region:*

*Obtain with:*

*=TINV(2\*α, n-1) Excel*

*abs(qt(α, n-1)) R*

*P-value: upper tail area*

*Obtain P-value with:*

*=TDIST(t, n-1, 1) Excel*

*1-pt(t, n-1, FALSE) R*

1. **Ha: *lower-tailed test***

*Rejection Region:*

*Obtain with:*

*=(TINV(2\*α, n-1))\*(-1) Excel*

*qt(α, n-1) R*

*P-value: lower tail area*

*Obtain P-value with:*

*=TDIST(ABS(t), n-1, 1) Excel*

*pt(-1\*abs(t), n-1, FALSE) R*

1. **Ha: *two-tailed test***

*Rejection Region:*  OR

*Obtain with:*

*=TINV(α, n-1) Excel*

*abs(qt(α/2, n-1, FALSE)) R*

*P-Value: area in 2 tails*

*Obtain P-value with:*

*=TDIST(ABS(t), n-1, 2) Excel*

*2\*(1-pt(abs(t),n-1)) R*